



Process Systems Engineering and Process Safety

An Approach to Continuous Approximation of Pareto Front Using Geometric Support Vector Regression for Multi-objective Optimization of Fermentation Process[☆]

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ABSTRACT

The approaches to discrete approximation of Pareto front using multi-objective evolutionary algorithms have the problems of heavy computation burden, long running time and missing Pareto optimal points. In order to overcome these problems, an approach to continuous approximation of Pareto front using geometric support vector regression is presented. The regression model of the small size approximate discrete Pareto front is constructed by geometric support vector regression modeling and is described as the approximate continuous Pareto front. In the process of geometric support vector regression modeling, considering the distribution characteristic of Pareto optimal points, the separable augmented training sample sets are constructed by shifting original training sample points along multiple coordinated axes. Besides, an interactive decision-making (DM) procedure, in which the continuous approximation of Pareto front and decision-making is performed interactively, is designed for improving the accuracy of the preferred Pareto optimal point. The correctness of the continuous approximation of Pareto front is demonstrated with a typical multi-objective optimization problem. In addition, combined with the interactive decision-making procedure, the continuous approximation of Pareto front is applied in the multi-objective optimization for an industrial fed-batch yeast fermentation process. The experimental results show that the generated approximate continuous Pareto front has good accuracy and completeness. Compared with the multi-objective evolutionary algorithm with large size population, a more accurate preferred Pareto optimal point can be obtained from the approximate continuous Pareto front with less computation and shorter running time. The operation strategy corresponding to the final preferred Pareto optimal point generated by the interactive DM procedure can improve the production indexes of the fermentation process effectively.

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1. Introduction

Fermentation is the basis of bioengineering, modern biotechnology and their industrialization [1]. With the rapid development of the fermentation technology and the continuous expansion of production scale of fermentation industry, it is urgent to execute process optimization and control to improve the production quality and economic benefit of fermentation process [2,3]. The early single objective optimization for fermentation process cannot deal with a number of production indexes such as production output, substrate cost, and fermentation time in fermentation process at the same time [4]. The Multi-Objective Optimization (MOO) for fermentation process is an effective way to improve the production quality and benefits of fermentation process [5,6].

The existing methods for MOO of fermentation process using linear weighted summation need the artificial weights of objectives which may have a negative impact on the results of optimization. Therefore, these methods are only appropriate for the case that the objectives are not competing with each other [7,8]. Adopting Multi-Objective Evolutionary Algorithms (MOEAs) [9,10] to generate approximate discrete Pareto front combined with using Decision-Making (DM) algorithms [11,12] to select the preferred Pareto optimal point is a common way to deal with MOO problem [13,14]. However, MOEAs require large size population to search Pareto optimal points and need a great number of evolutionary generations, which often lead to substantial evaluations of objective functions. Then, MOEAs often have the problems of heavy computation burden and long running time and cannot deal with the MOO problem involving large-scale data in fermentation process. In order to reduce computation burden, some methods seek to limit the search in an interested region instead of the complete search space by introducing the preference information before or during the search of Pareto optimal points [15,16]. However, because a complete Pareto front cannot be

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provided, some important Pareto optimal points located outside the interested region may be left out.

In this paper, an approach to continuous approximation of Pareto front using geometric Support Vector Regression (SVR) is presented. The regression model of the small size approximate discrete Pareto front is constructed by geometric SVR modeling and is described as the approximate continuous Pareto front. An interactive DM procedure, in which the continuous approximation of Pareto front and DM is performed interactively, is designed for improving the accuracy of the preferred Pareto optimal point. The correctness of the continuous approximation of Pareto front is demonstrated with a typical multi-objective optimization problem. In addition, combined with the interactive DM procedure, the continuous approximation of Pareto front is applied in the multi-objective optimization for an industrial fed-batch yeast fermentation process.

2. An Approach to Continuous Approximation of Pareto Front Using Geometric SVR

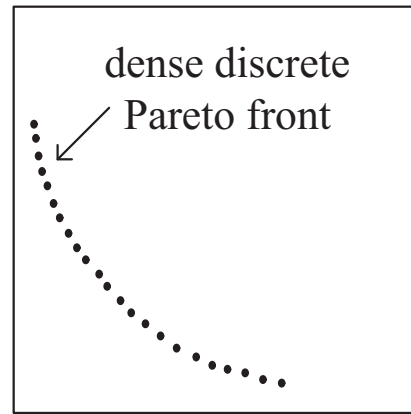
2.1. Principle of continuous approximation of Pareto front

The principle of the presented approach to continuous approximation of Pareto front can be described as follows. At first, a complete and sparse approximate discrete Pareto front is obtained by the MOEA with small size population. Although the generated discrete Pareto front includes only a few Pareto optimal points and has blind area between the Pareto optimal points, it can provide relatively complete information of the coverage area of the true Pareto front in the search space because of its good distribution. Thus, the problem of missing Pareto optimal points can be avoided effectively. Moreover, the computation burden is reduced because of the reduction of the population size. After that, the regression model of the generated approximate discrete Pareto front is constructed by geometric SVR modeling and is described as the approximate continuous Pareto front. Then, the blind area between the Pareto optimal points can be eliminated as shown in Fig. 1.

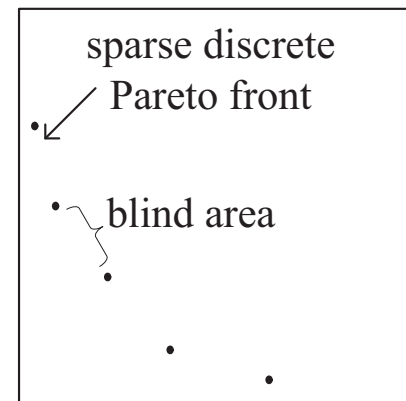
2.2. Geometric SVR modeling

The Geometric SVR modeling is adopted to generate the approximate continuous Pareto front. Due to avoiding the quadratic programming problem, geometric SVR has less computation and higher efficiency than standard SVR [17]. Vapnik [18] presented using ε -insensitive loss function to construct regression model and an example was in an error if its residual $|y - f(x)|$ was greater than ε ($0 < \varepsilon \ll 1$). In the geometric interpretation, ε -insensitive loss function can be described as ε -tube [19]. Namely, for planar case, an accurate regression model means that all training sample points should fall in a ε -tube which is formed by shifting the regression function up and down by ε along y axis. According to ε -tube, Bi and Bennett [19] presented that geometric SVR problem can be transformed into the Nearest Point Problem (NPP) by augmenting training sample set. Let $T = \{(\mathbf{x}_i, y_i) : \mathbf{x}_i \in \mathbf{R}^m, y_i \in \mathbf{R}, i = 1, 2, \dots, n\}$ be a training sample set, where $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n]$ is predictor vector and $\mathbf{y} = [y_1, y_2, \dots, y_n]$ is response vector. The augmented training sample sets $\mathbf{D}^+$ and $\mathbf{D}^-$ can be constructed by shifting the original training sample points with the response variables increased and decreased by $\hat{\varepsilon}$, respectively. That is

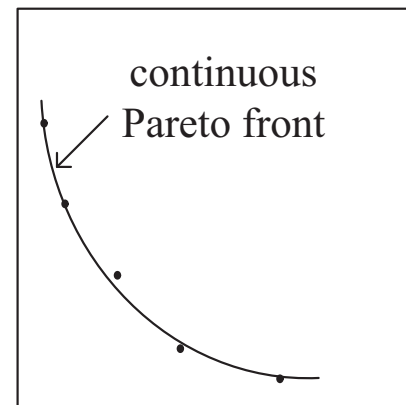
$$\begin{cases} \mathbf{D}^+ = \left(\begin{matrix} \mathbf{X}^T \\ (\mathbf{y} + \hat{\varepsilon} \mathbf{I}_n)^T \end{matrix} \right)^T \\ \mathbf{D}^- = \left(\begin{matrix} \mathbf{X}^T \\ (\mathbf{y} - \hat{\varepsilon} \mathbf{I}_n)^T \end{matrix} \right)^T \end{cases} \quad (1)$$



(a) Large size approximate discrete Pareto front



(b) Small size approximate discrete Pareto front



(c) Approximate continuous Pareto front

Fig. 1. Principle of continuous approximation of Pareto front.

Two Convex Hulls (CH) $\text{Co}(\mathbf{D}^+)$ and $\text{Co}(\mathbf{D}^-)$ can be constructed based on the convex combinations of the points in $\mathbf{D}^+$ and $\mathbf{D}^-$, respectively. Then, the NPP between the two CHs is

$$\begin{cases} \min J(\boldsymbol{\alpha}, \boldsymbol{\beta}) = \frac{1}{2} \left\| \boldsymbol{\alpha}^T \begin{pmatrix} \mathbf{X}^T \\ (\mathbf{y} + \hat{\varepsilon} \mathbf{I}_n)^T \end{pmatrix} - \boldsymbol{\beta}^T \begin{pmatrix} \mathbf{X}^T \\ (\mathbf{y} - \hat{\varepsilon} \mathbf{I}_n)^T \end{pmatrix} \right\|^2 \\ \text{s.t. : } \mathbf{I}_n^T \boldsymbol{\alpha} = \mathbf{I}_n^T \boldsymbol{\beta} = 1, 0_n \leq \boldsymbol{\alpha} \leq \mathbf{I}_n, 0_n \leq \boldsymbol{\beta} \leq \mathbf{I}_n \end{cases} \quad (2)$$

where $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ and $\beta = (\beta_1, \beta_2, \dots, \beta_n)$ are the Lagrange multipliers of two augmented training sample sets. According to ε -tube, the optimal separating hyperplane obtained by solving the NPP between two CHs of D^+ and D^- is described as the regression function of the original training sample points. It can be formulated as

$$\begin{cases} f(\mathbf{x}) = \sum_{i=1}^n (\bar{\beta}_i^* - \bar{\alpha}_i^*) K(\mathbf{x}_i, \mathbf{x}) + \bar{b}^* \\ \text{s.t. :} \\ \bar{\alpha}_i^* = \frac{\alpha_i^*}{\delta}, \bar{\beta}_i^* = \frac{\beta_i^*}{\delta}, \delta = \sum_{i=1}^n (\alpha_i^* - \beta_i^*) y_i + 2\hat{\varepsilon} \\ \bar{b}^* = \frac{\sum_{i=1}^n (\alpha_i^* - \beta_i^*) K(\mathbf{x}_i, \mathbf{x}) (\alpha_i^* + \beta_i^*)}{2\delta} + \frac{\sum_{i=1}^n (\alpha_i^* + \beta_i^*) y_i}{2} \end{cases} \quad (3)$$

where, $K(\cdot)$ is a kernel function, $\alpha^* = [\alpha_1^*, \alpha_2^*, \dots, \alpha_n^*]$ and $\beta^* = [\beta_1^*, \beta_2^*, \dots, \beta_n^*]$ are the optimal solutions of the NPP (2). Fig. 2 illustrates the process of geometric SVR modeling.

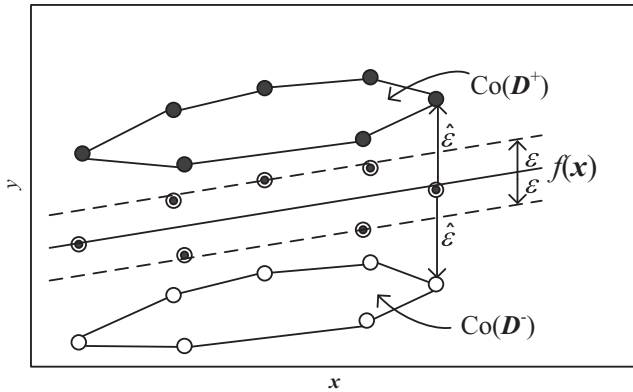


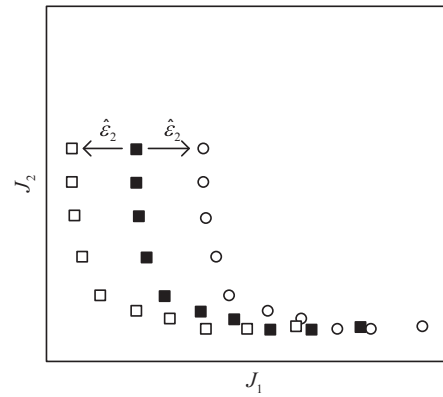
Fig. 2. Geometric SVR modeling.

2.3. Continuous approximation of Pareto front using geometric SVR

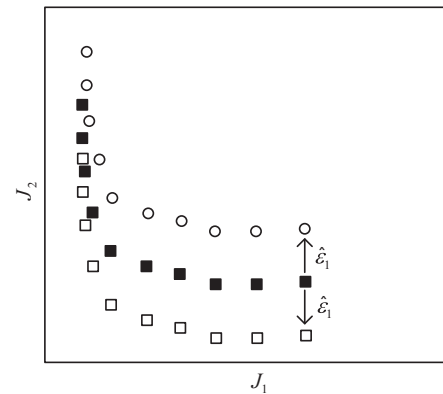
The continuous approximation of Pareto front can be achieved by constructing the geometric SVR model of Pareto optimal points. Because there are usually some Pareto optimal points parallel to one or more coordinate axes, it is hard to construct two separable augmented training sample sets by shifting the original training sample points only along one coordinate axis, which may result in the problem that the obtained approximate continuous Pareto front cannot cover all Pareto optimal points as shown in Fig. 3. Therefore, considering the distribution characteristic of Pareto optimal points, the augmented training sample sets are constructed by shifting the original training sample points along multiple coordinated axes as shown in Fig. 4.

Let the number of objective functions be m ; $\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_n]$ ($\mathbf{p} \in \mathbf{R}^m$) is the set of Pareto optimal points. Two augmented training sample sets $\bar{\mathbf{P}}^+$ and $\bar{\mathbf{P}}^-$ can be constructed by shifting $\mathbf{P}$ along each coordinated axis in the space of objective functions.

$$\begin{cases} \bar{\mathbf{P}}^+ = [\mathbf{p}_1 + \hat{\varepsilon}, \mathbf{p}_2 + \hat{\varepsilon}, \dots, \mathbf{p}_n + \hat{\varepsilon}] \\ \bar{\mathbf{P}}^- = [\mathbf{p}_1 - \hat{\varepsilon}, \mathbf{p}_2 - \hat{\varepsilon}, \dots, \mathbf{p}_n - \hat{\varepsilon}] \\ \hat{\varepsilon} = [\hat{\varepsilon}_1, \hat{\varepsilon}_2, \dots, \hat{\varepsilon}_m] \end{cases} \quad (4)$$



(a) Shifting original sample points along J_1



(b) Shifting original sample points along J_2

Fig. 3. Construction of augmented sample sets by shifting original sample points along single coordinated axis.

The NPP between the two CHs $\text{Co}(\bar{\mathbf{P}}^+)$ and $\text{Co}(\bar{\mathbf{P}}^-)$ can be represented as

$$\begin{cases} \min J(\alpha, \beta) = \frac{1}{2} \|\alpha^T \bar{\mathbf{P}}^+ - \beta^T \bar{\mathbf{P}}^-\| \\ \text{s.t. : } \mathbf{I}_n^T \alpha = \mathbf{I}_n^T \beta = 1, 0_n \leq \alpha \leq \mathbf{I}_n, 0_n \leq \beta \leq \mathbf{I}_n \end{cases} \quad (5)$$

Then, the geometric SVR model of Pareto optimal points can be formulated as

$$f(\mathbf{p}) = [\alpha^{*T}, \beta^{*T}] K([\bar{\mathbf{P}}^+, -\bar{\mathbf{P}}^-], \mathbf{p}) + b^* \quad (6)$$

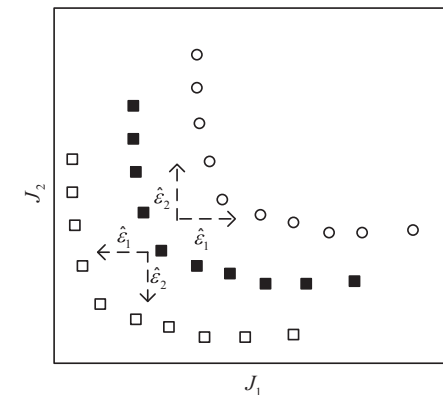


Fig. 4. Construction of augmented sample sets by shifting original sample points along multiple coordinated axes.

2.4. Test case for continuous approximation of Pareto front

A test case concerning a typical MOO problem is introduced to demonstrate the correctness of the continuous approximation of Pareto front. The objective functions presented by Deb [20] are expressed as

$$\begin{cases} \min J_1(x_1, x_2) = x_1 \\ \min J_2(x_1, x_2) = \left(2 - e^{-\frac{(x_2 - 0.2)^2}{0.004}} - 0.8e^{-\frac{(x_2 - 0.6)^2}{0.04}}\right) \frac{1}{x_1} \end{cases} \quad (7)$$

where $0.1 \leq x_1, x_2 \leq 1$. SEC-MOPSO algorithm [21] is adopted to search Pareto optimal points. The parameters of SEC-MOPSO with small size population are set as: the size of population $m = 20$, the number of evolutionary generations is 100, inertia weight $\omega = 1$, acceleration coefficient $c_1 = c_2 = c_3 = c_4 = 0.1$ and swarming coefficient is 0.7. The parameters of geometric SVR are set as: the parameter of Gaussian kernel function $\sigma = 0.1$ and the parameter of insensitive loss function $\varepsilon = 0.01$. Two pairs of augmented training sample sets are constructed by shifting the original training sample points along single coordinated axis (the shift parameter is 0.9 along J_2) and multiple coordinated axis (the shift parameters are 0.16 along J_1 and 1.2 along J_2), respectively. Gilbert's algorithm [17] is also adopted to solve the NPP between the two CHs constructed based on the two augmented training sample sets. Besides, a large size approximate discrete Pareto front generated by SEC-MOPSO with large size population ($m = 200$) is also introduced to evaluate the performance of approximate continuous Pareto front. Other parameters of the SEC-MOPSO with large size population are same as the parameters of the SEC-MOPSO with small size population. Figs. 5 and 6 show the approximate continuous Pareto front generated by geometric SVR modeling. It can be seen from Fig. 5 that because there are some Pareto optimal points parallel to J_2 , two separable augmented training sample sets cannot be constructed by shifting the original training sample points only along J_2 and the obtained approximate continuous Pareto front cannot cover all Pareto optimal points. Fig. 6 shows that shifting the original training sample points along J_1

and J_2 can construct two separable augmented training sample sets. Thus, the generated approximate continuous Pareto front has good accuracy and completeness.

3. Procedure of Interactive DM Involving Continuous Approximation of Pareto Front

3.1. Principle of interactive DM procedure

Because the approximate continuous Pareto front is generated by geometric SVR modeling instead of MOEA, the accuracy of the preferred Pareto optimal point selected from the approximate continuous Pareto front cannot be guaranteed. Therefore, an interactive DM procedure involving the continuous approximation of Pareto front is designed for improving the accuracy of the approximate continuous Pareto front and the preferred Pareto optimal point. The principle of the procedure can be described as follows. A preferred Pareto optimal point is selected from the generated complete approximate continuous Pareto front by DM algorithm at first. This process can be described as the first DM. Although the preferred Pareto optimal point obtained by the first DM may be not accurate because of the errors of the approximate continuous Pareto front, its neighborhood can be considered as an interested region where the Pareto optimal points are more suitable to the specified preference information. In addition, the completeness of the approximate continuous Pareto front can avoid missing Pareto optimal points. In order to improve the accuracy of the approximate continuous Pareto front and the preferred Pareto optimal point, the continuous approximation of Pareto front and DM will be performed again in the small neighborhood of the preferred Pareto optimal point obtained by the first DM. With the contraction of the interested region, the accuracy of the approximate continuous Pareto front and the preferred Pareto optimal point will be improved. The above procedure will be performed interactively until the result of DM can meet the accuracy requirement. The accuracy of the preferred Pareto optimal point can be measured based on the distance between the preferred Pareto optimal point

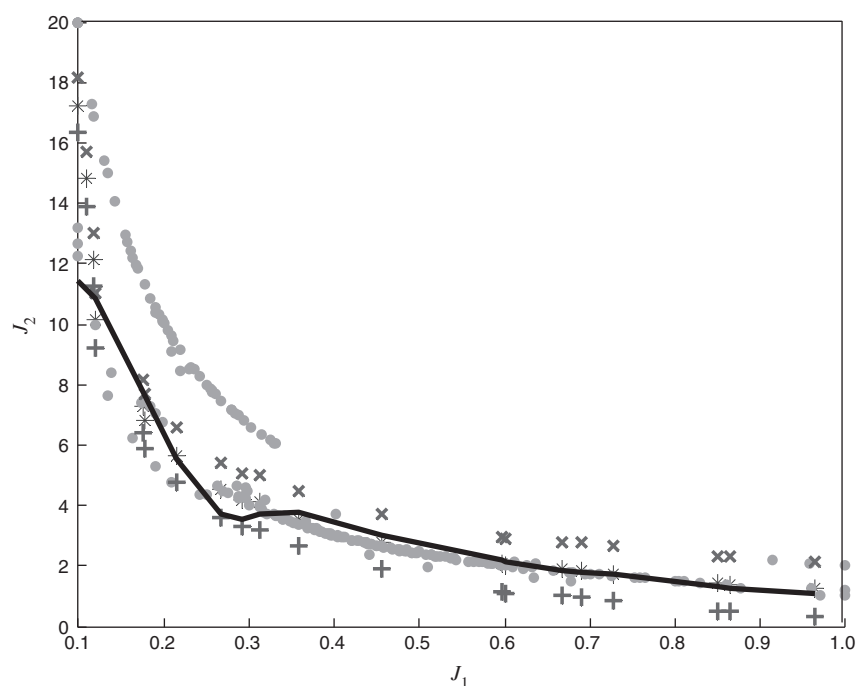


Fig. 5. Approximate continuous Pareto front generated by shifting original training sample points along J_2 . * small size discrete Pareto front; • large size discrete Pareto front; × augmented training sample points; + augmented training sample points; — approximate continuous Pareto front.

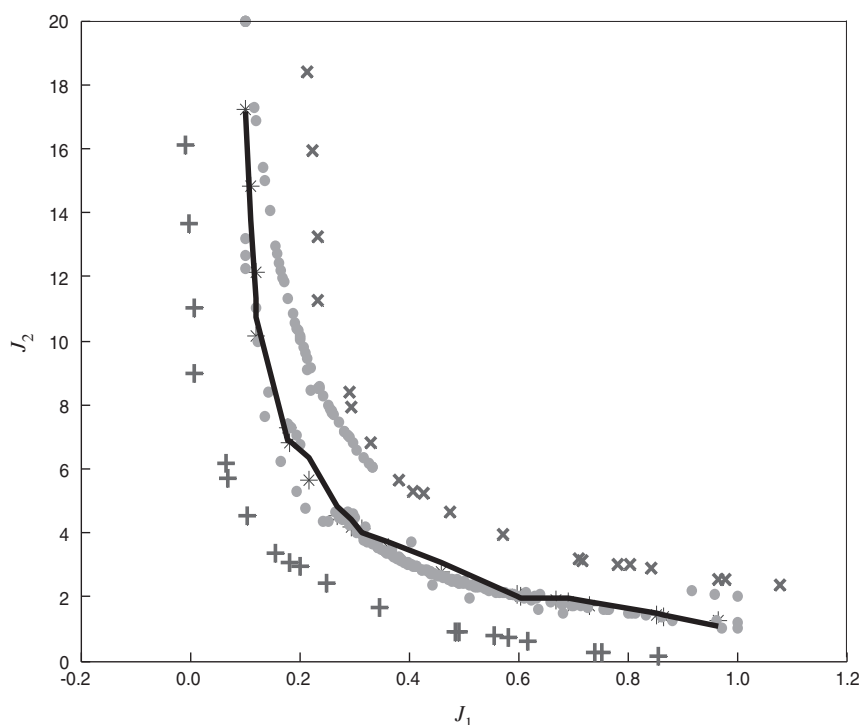


Fig. 6. Approximate continuous Pareto front generated by shifting original training sample points along J_1 and J_2 . * small size discrete Pareto front; • large size discrete Pareto front; × augmented training sample points; + augmented training sample points; — approximate continuous Pareto front.

selected from the approximate continuous Pareto front and its closest Pareto optimal point generated by MOEA. Let the number of objective functions be m and i^* is the preferred Pareto optimal point selected from the approximate continuous Pareto front. Then, the distance can be formulated as follows

$$d(i^*) = \sum_{k=1}^m \left| \frac{J_{k,i^*} - J_{k,\min_d}}{\bar{J}_{k,\max} - \bar{J}_{k,\min}} \right| \quad (8)$$

where $\bar{J}_{k,\max}$ and $\bar{J}_{k,\min}$ are the maximum value and minimum value of the k th objective function of Pareto optimal points obtained by the first DM; J_{k,i^*} is the value of k th objective function of i^* and $J_{k,\min_d}$ is the value of k th objective function of the nearest Pareto optimal point obtained by MOEA to i^* .

3.2. Implementation of interactive DM procedure

Implementation of the interactive DM procedure can be summarized in the following steps.

- Step 1 Initialization: Declare the objective functions and the constraint conditions of the MOO problem. Set the parameters that are involved in the MOEA, the continuous approximation of Pareto front and the DM algorithm.
- Step 2 Discrete approximation of Pareto front: Generate the approximate discrete Pareto front by the MOEA with small size population.
- Step 3 Continuous approximation of Pareto front: Construct the regression model of the generated approximate discrete Pareto front by geometric SVR modeling.
- Step 4 The first DM: Select the preferred Pareto optimal point from the approximate continuous Pareto front by the DM algorithm.
- Step 5 Set the interested region: Set the small neighborhood of the current preferred Pareto optimal point as the interested region.

The size of the interested region can be adjusted according to the accuracy of the preferred Pareto optimal point.

- Step 6 Discrete approximation of Pareto front in interested region: Generate the approximate discrete Pareto front in interested region by the MOEA with small size population.
- Step 7 Continuous approximation of Pareto front in the interested region: Construct the regression model of the generated approximate discrete Pareto front in the interested region by geometric SVR modeling.
- Step 8 DM in the interested region: Select the preferred Pareto optimal point from the approximate continuous Pareto front in the interested region by the DM algorithm.
- Step 9 Check the termination condition: End the loop if the termination condition is satisfied; otherwise, go to Step 5. The termination condition is that the index Eq. (8) meets the specified accuracy requirement.

4. MOO for Fed-batch Fermentation Process

4.1. MOO problem in fed-batch fermentation process

Combined with the interactive DM procedure, the continuous approximation of Pareto front is applied in an industrial fed-batch yeast fermentation process in order to find an optimal substrate feed rate profile that maximizes the final biomass and minimizes the total consumption of substrate. The total time of fermentation process can be separated equally into several intervals. Then, the substrate feed rate profile consists of the different substrate feed rates in each interval. The MOO problem can be formulated as

$$\begin{cases} \min J_1 = 1/(C_x(t_f)V(t_f)) \\ \min J_2 = \sum_{i=1}^d F_i \frac{t_f}{d} \\ \text{s.t. } F_{\min} \leq F \leq F_{\max} \end{cases} \quad (9)$$

where C_x ($\text{kg} \cdot \text{L}^{-1}$) is the concentration of biomass, V (L) is the culture volume, F ($\text{L} \cdot \text{h}^{-1}$) is the substrate feed rate, t_f is the terminal time, d is the number of intervals and the substrate feed rate profile can be described as a vector $\mathbf{F} = [F_1, F_2, \dots, F_d]$. J_1 and J_2 represent the reciprocal of final biomass and total consumption of substrate, respectively.

4.2. Results and discussion

The industrial fed-batch yeast fermentation process is simulated based on the kinetic model [22] and the dynamic model [4]. The total fermentation time for simulation is 16.5 h, which is equally separated into 10 intervals ($d = 10$). The samples during the total fermentation time are 100 from 0 to 16.5 h. The model parameters are set according to reference [4]. According to reference [7], the initial conditions including initial biomass concentration ($C_x/\text{kg} \cdot \text{L}^{-1}$), initial substrate concentration ($C_s/\text{kg} \cdot \text{L}^{-1}$), initial carbon dioxide concentration ($C_c/\text{kg} \cdot \text{L}^{-1}$), initial dissolved oxygen concentration ($C_o/\text{kg} \cdot \text{L}^{-1}$), initial ethanol concentration ($C_e/\text{kg} \cdot \text{L}^{-1}$) and initial culture volume (V/L) are listed in Table 1. The substrate feed rate F is limited within ($500\text{--}1200$) $\text{L} \cdot \text{h}^{-1}$.

Table 1
Initial conditions for dynamic simulation

$C_x/\text{kg} \cdot \text{L}^{-1}$	$C_s/\text{kg} \cdot \text{L}^{-1}$	$C_c/\text{kg} \cdot \text{L}^{-1}$	$C_o/\text{kg} \cdot \text{L}^{-1}$	$C_e/\text{kg} \cdot \text{L}^{-1}$	V/L
8×10^{-3}	7×10^{-3}	0	7.712×10^{-6}	0	5×10^{-4}

4.2.1. The first DM

In the first place, a small size approximate discrete Pareto front is generated by SEC-MOPSO. The parameters of SEC-MOPSO with small size population are set as: the size of population $m = 50$, the number of evolutionary generations is 100, inertia weight $\omega = 1$, acceleration coefficient $c_1 = c_2 = c_3 = c_4 = 0.1$ and swarming coefficient is 0.7. Secondly, an approximate continuous Pareto front is generated by

geometric SVR modeling. The parameters of geometric SVR are set as: the parameter of Gaussian kernel function $\sigma = 0.1$ and the parameter of insensitive loss function $\varepsilon = 0.01$. The values of objective functions are normalized as $J_1 = J_1 \times 10^4$ and $J_2 = J_2 \times 10^{-4}$. Then, the shift parameters of augmented training sample sets are 0.1 along J_1 and 0.05 along J_2 . A classical Multi-Attribute Decision-making (MADM) algorithm named TOPSIS [13] is introduced to select the preferred Pareto optimal point from the approximate continuous Pareto front. In TOPSIS, the weighted parameters of J_1 and J_2 are both set as 0.5 according to the preference information. The upper limit value of accuracy index Eq. (8) is 0.01. A large size approximate discrete Pareto front generated by SEC-MOPSO with large size population ($m = 200$) is also introduced to evaluate the performance of approximate continuous Pareto front and the results of DM. Other parameters of the SEC-MOPSO with large size population are the same as the parameters of the SEC-MOPSO with small size population. Fig. 7 shows the results of the first DM.

As shown in Fig. 7, the small size approximate discrete Pareto front has good distribution. Therefore, the generated approximate continuous Pareto front can provide relatively complete information of the coverage area of the large size approximate discrete Pareto front. The root-mean-square error (RMSE) of the approximate continuous Pareto front is 0.054. The accuracy index Eq. (8) of the first DM is 0.014 which is above the upper limit value. In order to improve the accuracy of the preferred Pareto optimal point, the above procedure is performed again in the small neighborhood of the preferred Pareto optimal point. The size of the neighborhood is set as $[-0.1 \times 10^4, 0.1 \times 10^4]$ along J_2 . Other parameters of the involved algorithms are the same as the parameters in the first DM. Fig. 8 shows the results of the DM in the interested region. The RMSE of the approximate continuous Pareto front in the interested region is 0.0035. The accuracy index Eq. (8) is 0.0032 which meets the accuracy requirement. Moreover, it can be seen from Fig. 8 that the final preferred Pareto optimal point is closer to the original point than all Pareto optimal points generated by SEC-MOPSO with large size population, which means the preferred Pareto optimal point has better accuracy in terms of the values of the objective functions.

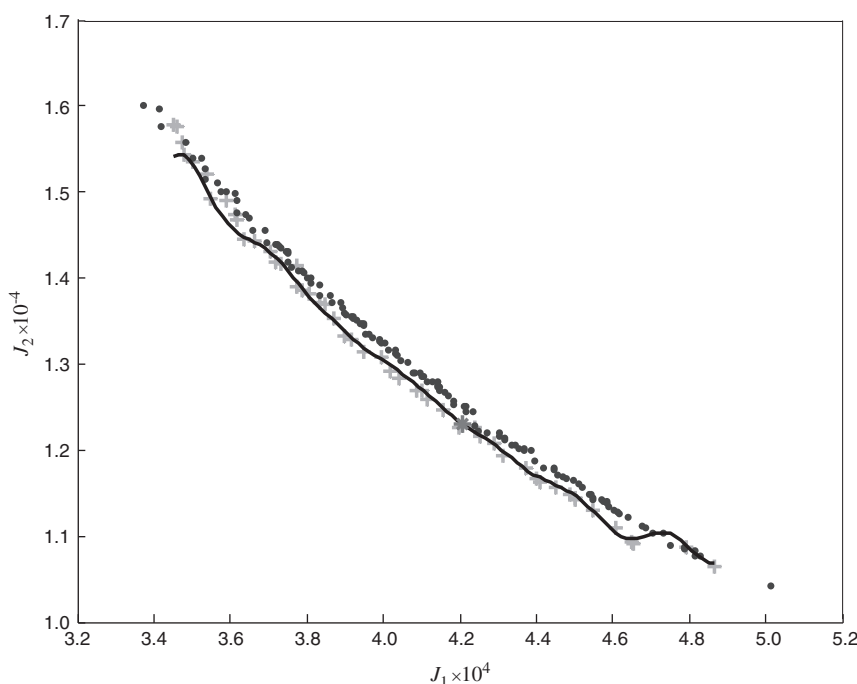


Fig. 7. Results of the first DM. + small size discrete Pareto front; • large size discrete Pareto front; — approximate continuous Pareto front; * result of DM.

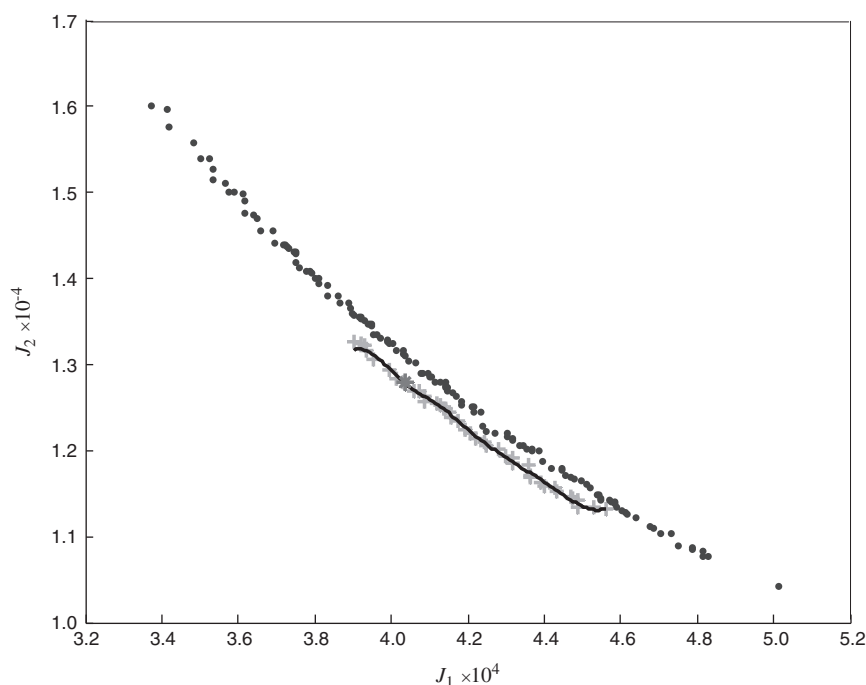


Fig. 8. Results of the DM in interested region. + small size discrete Pareto front; • large size discrete Pareto front; — approximate continuous Pareto front; • result of DM.

The comparison of computation efficiency between the interactive DM procedure and SEC-MOPSO with large size population is listed in Table 2, in which the number of evaluations of the objective functions (Obj) and the running time (time/h) are considered. Table 2 illustrates that the designed interactive DM procedure involving continuous approximation of Pareto front requires less computation and shorter running time.

Table 2
Comparison of computation efficiency

	Obj	Time/h
SEC-MOPSO ($m = 200$)	40000	9.62
Interactive DM procedure	20000	4.89

The optimal substrate feed rate profile (F_1) corresponding to the final preferred Pareto optimal point is implemented in the simulation of an industrial fed-batch yeast fermentation process and compared with two other common substrate feed rate profiles (constantly and exponentially feed rate profile) which are set as $F_2 = \{F_{2i} : F_{2i} = 800\}$ and $F_3 = \{F_{3i} : F_{3i} = 500e^{0.05t}\}$ ($i = 1, \dots, d$). The three substrate feed rate profiles and the profiles of key variables in the fermentation process are shown in Fig. 9.

In industrial fed-batch yeast fermentation process, the ethanol concentration should be minimized at the end of the process because the yield and quality of the final product will be deteriorated in the case of the excessive formation of ethanol [22]. Fig. 9 shows that the three substrate feed rate profiles can suppress the formation of ethanol effectively. Specifically, the optimal substrate feed rate profile (F_1) corresponding to the final preferred Pareto optimal point and the exponentially feed rate profile (F_3) have better performance in suppressing the

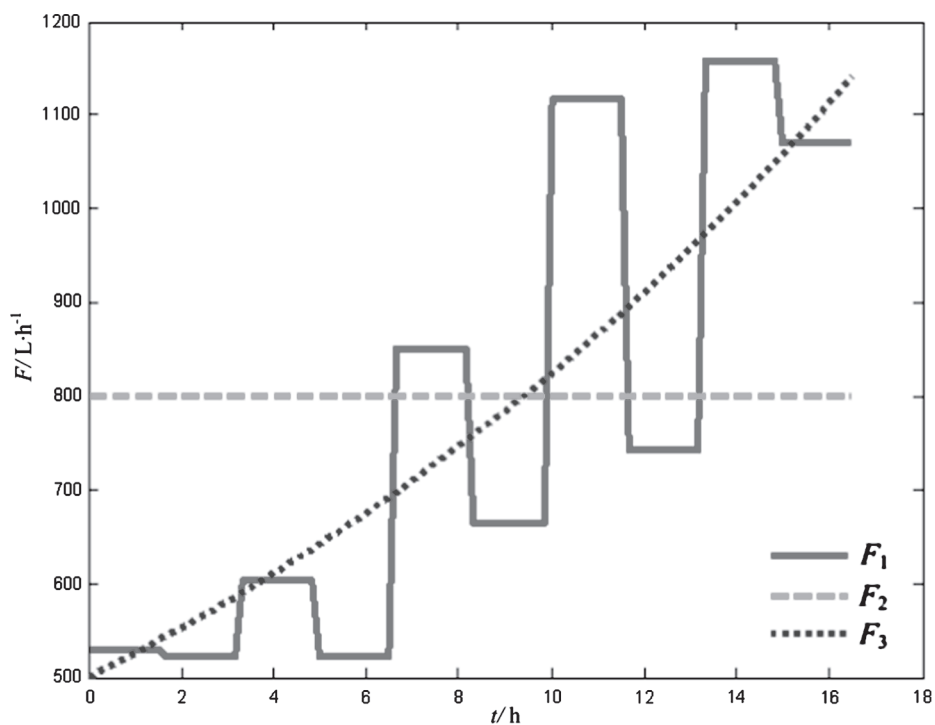
formation of ethanol. Two production indexes of fermentation process corresponding to the three substrate feed rate profiles are listed in Table 3, in which the final biomass ($1/J_1$) and the total consumption of substrate (J_2) are considered. As seen in Table 3, the substrate feed rate profile corresponding to the final preferred Pareto optimal point can obtain more final biomass with less total consumption of substrate than the two other feed rate profiles.

5. Conclusions

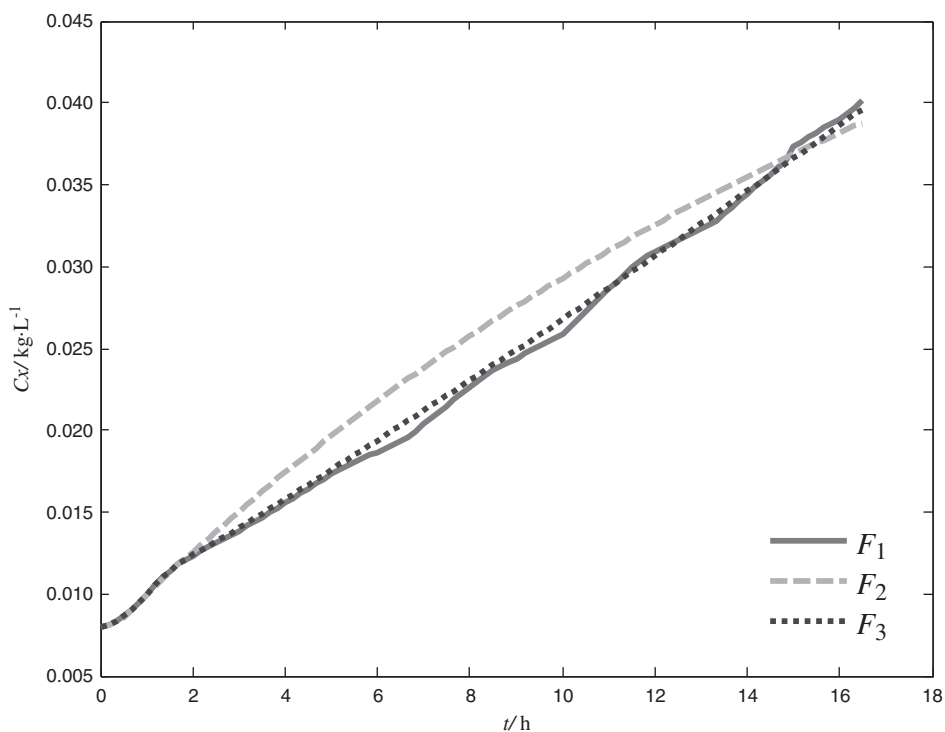
An approach to continuous approximation of Pareto front using geometric SVR is presented. The regression model of the small size approximate discrete Pareto front is constructed by geometric SVR modeling and is described as the approximate continuous Pareto front. An interactive DM procedure, in which the continuous approximation of Pareto front and DM is performed interactively, is also designed for improving the accuracy of the preferred Pareto optimal point. The correctness of the continuous approximation of Pareto front is demonstrated with a typical multi-objective optimization problem. In addition, combined with the interactive DM procedure, the continuous approximation of Pareto front is applied in the multi-objective optimization for an industrial fed-batch yeast fermentation process. The experimental results show that shifting original training sample points along multiple coordinated axes can construct separable augmented training sample sets so that the generated approximate continuous Pareto front has better accuracy and completeness. Furthermore, compared with the MOEA with large size population, a more accurate preferred Pareto optimal point can be obtained by the continuous approximation of Pareto front and the interactive DM procedure with less computation and shorter running time. The operation strategy corresponding to the final preferred Pareto optimal point generated by the interactive DM procedure can obtain more final biomass with less total consumption of substrate which means that the production indexes of the fermentation process are improved effectively.

Nomenclature

b	offset	C_x	biomass concentration, $\text{kg} \cdot \text{L}^{-1}$
C_C	carbon dioxide concentration, $\text{kg} \cdot \text{L}^{-1}$	c	acceleration coefficient
C_E	ethanol concentration, $\text{kg} \cdot \text{L}^{-1}$	$\mathbf{D}$	augmented training sample
C_O	dissolved oxygen concentration, $\text{kg} \cdot \text{L}^{-1}$	d	number of intervals
$\text{Co}(\cdot)$	convex combination	$\bar{d}(\cdot)$	accuracy index of DM
C_S	substrate concentration, $\text{kg} \cdot \text{L}^{-1}$	F	substrate feed rate, $\text{L} \cdot \text{h}^{-1}$
		$f(\cdot)$	regression function

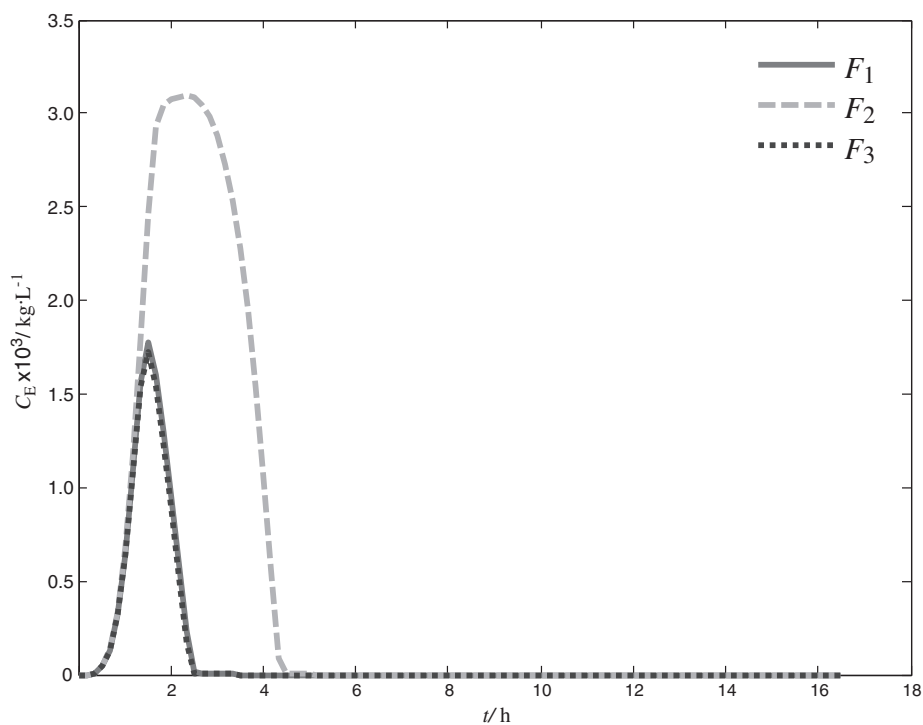


(a) Substrate feed rate profile

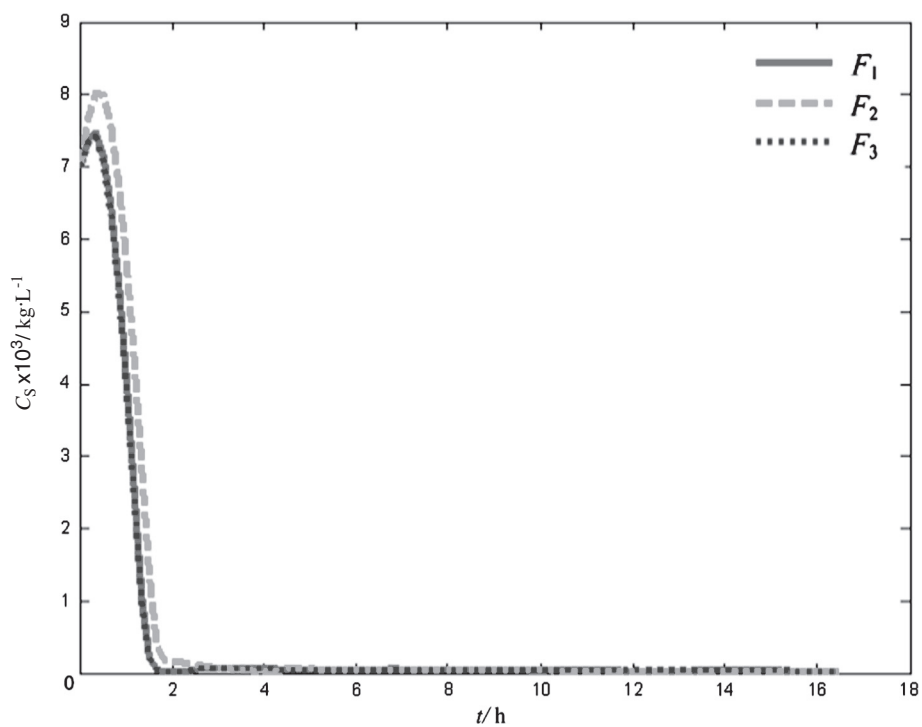


(b) Biomass concentration profile

Fig. 9. Profiles of key variables in industrial fed-batch yeast fermentation process.



(c) Ethanol concentration profile



(d) Glucose concentration profile

Fig. 9 (continued).

Table 3
Comparison of production indexes

	J_1^{-1}/kg	J_2/L
F_1	2503.4	12783
F_2	2452	13200
F_3	2486.6	12826

$\mathbf{I}$	unit vector
J	objective function
$K(\cdot)$	kernel function
m	population size
$\mathbf{p}$	Pareto optimal point
$\mathbf{R}$	real number field
$\mathbf{T}$	training sample set

t	time, h
t_f	terminal time
V	culture volume, L
$\mathbf{x}$	predictor vector
$\mathbf{y}$	response vector
α, β	Lagrange multiplier
ε	parameter of insensitive loss function
$\hat{\varepsilon}$	shift parameter
σ	parameter of Gaussian kernel function
ω	inertia weight

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